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OPTIMAL ORDERING POLICIES OF A SINGLE ITEM INVENTORY MODEL WITH STOCK- AND PRICE DEPENDENT DEMAND FOR DETERIORATING ITEMS WITH VARIABLE CYCLE LENGTHS

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ABSTRACT

Many goods undergo deterioration over time, which suffers from depletion by direct spoilage while in storage. So, decay or deterioration of these goods in stock is a very realistic feature and it is necessary to use this factor in inventory models. In the present paper, an inventory replenishment model for deteriorating items is developed with the assumptions that demand is a function of selling price and on-hand inventory. The cycle length of successive replenishments is a variable in the planning period. It is assumed that the cycle length in each cycle decreases in Arithmetic Progression. Shortages are allowed and are fully backlogged. The instantaneous state of inventory with shortages is derived. The total cost function of the horizon is obtained with suitable costs. The optimal pricing and ordering policies of the model are derived. The objective is to determine a replenishment policy that minimizes the total inventory cost. The model is illustrated with some numerical results. The sensitivity of the model with respect to the parameters and cost is also discussed. This model includes some of the earlier models as particular cases.

***KEYWORDS:** Cycle Length, Demand Rate and Cost Function, Optimal Ordering Policies, Perishable Inventory, Sensitivity Analysis*

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1. INTRODUCTION

Most of the physical goods undergo decay or deterioration over time. Research in the area of decaying or deteriorating items is important, because in real life items like milk, blood, drugs, fruits, vegetables, foodstuffs, etc. suffer from depletion by direct spoilage while kept in storage. Highly volatile liquids like gasoline, alcohol, turpentine etc. undergo physical depletion over time through the process of evaporation. Many researchers have studied the effect of lifecycle of goods under consideration on the inventory control. The first to mention is Ghare and Schrader [4] who developed the model for an item with an exponentially decaying inventory. Since then researches in deteriorating items have become very popular. Covert and Philip [3] extended Ghare and Schrader's [4] model and obtained an economic order quantity model for a variable rate of deterioration by assuming a two-parameter Weibull distribution. Rafat, F [11] made an extensive survey of literature on continuously deteriorating inventory model. Aggarwal, S. P. [1] presented an order level inventory model for deteriorating items with a constant rate of deterioration, which had been developed both for deterministic and stochastic demands. Shah [15] was the first to consider deterioration with complete backlogging. Nahimias [9], Giri and Goel [5] have reviewed the Inventory models of deteriorating items. Miguel F. Anjos, Russell C. H. Cheng & Christine S. M. Currie [8] proposed optimal pricing policies for perishable products Jian L et al.[7] gave an analysis of postponement

strategy for perishable items by EOQ-based models. Roy and Chauduri[12], have developed an EOQ model for deteriorating items with price dependent demand and special sales. Sana. S. S. [14] studied an EOQ model for seasonal goods. Roy [13] developed a deterministic inventory model when the deterioration rate is time proportional. Demand rate is a function of selling price, and holding cost is time dependent. Panda, Senapathi and Basu[10] have developed a single cycle perishable inventory model with time dependent quadratic ramp type demand and partial backlogging. They assumed that deterioration of inventory starts after a certain time. Tripathy and Mishra [16] discussed a model for deteriorating items with price dependent demand and linear holding cost. Chun-Tao Chang et al[2] developed and anlysed inventory models with stock and price dependent demand for deteriorating items based on limited shelf space Roy and Chaudur [12]i, (2010); have studied an inventory model for a perishable item assuming that the demand rate depend not only on time but also on the selling price of the item. Jayasree. P. R [6] have studied an inventory model for deteriorating items with variable type Demand rate and different selling prices. Yadav Smita et al [17] considered an inventory model in which demand is taken as a function of selling price and variable rate of deterioration is taken as a linear function of time and a storage time dependent holding cost. The holding cost per unit of the item is considered to be a linear function of time spent in storage.

2. NOVELTY OF THE WORK

Recently, much emphasis has been given in literature regarding inventory for deteriorating items with various types of demand and replenishment. The potential strings in inventory modeling are making suitable assumptions for the demand and the cycle length (the time laps between two successive replenishments). Several of the earlier researchers considered that the cycle length is fixed and constant for each cycle of planning period in the finite horizon. This assumption is useful only when we consider a single period EQQ models with non-deteriorating items. However, for the perishable items like fruits, food grains, edible oils, paints, chemicals, sea foods, cement etc., the item is subject to deterioration due to various facts like temperature, humidity, chemical reactions, vaporization etc., over the storage period. The planning is made for the entire horizon having different number of cycles. The cycle length is tremendously influenced by various operating conditions and has an influence on operating policies. Hence it is needed to consider variable cycle lengths for each cycle in the entire planning period (horizon).

It is a common belief that for certain items the demand is influenced by the stock on hand and selling price. For example this phenomenon is visualised in inventory systems at market yards, edible oil plants, food and vegetable markets, chemical industries where depending on the selling price, the demand fluctuates. The selling price is one of the decisive factors for selecting an item for use. It is well known that lesser the selling price of an item, increases the demand of that item, where as higher selling price will have a reverse effect. Hence, the demand can be viewed as a function of the selling price. t is also assumed that the life time of the commodity is random and follows an exponential distribution. The exponential distribution characterises for constant rate of deterioration. It is further assumed that shortages are allowed and fully backlogged.

3. ASSUMPTIONS AND NOTATIONS

The assumptions and notations used for developing the model are:

- The selling price per unit be 's' and known.

- The demand rate at any instant 't' is a linear function of 's' and onhand inventory $I_i(t)$ and is of the form $\lambda(s,t)=(\alpha-\beta s)+\gamma I_i(t)$
- Replenishment is instantaneous.
- Let the planning horizon be H units of time and is known. Inventory level is zero at times t = 0 and t = H.
- Lead time is zero.
- Shortages are allowed and fully backlogged, and shortages are not allowed in the last cycle.
- T_i is the total time elapsed upto and including the i^{th} cycle (i=1,2,..., m), where 'm' denotes the total number of replenishments to be made during the prescribed time hoizon H. Hence T_0=0 and T_m=H.
- t_i is the time at which the inventory level in the $(i-1)^{th}$ cycle reaches to zero (i=1,2,...,(m-1).
- The is the length of the first replenishment cycle and 'w' is the rate of reduction in the successive cycle lengths.
- The onhand inventory deteriorates at constant rate θ per unit time and there is neither repair nor replenishment of the deteriorated items during 'H'.
- Let the inventory holding cost 'C_1' per unit per unit time, shortage cost 'C_2' per unit per unit time, unit cost 'C' and the replenishment cost (ordering cost) be C_3 per replenishment are known and constant during the planning horizon 'H'.

4. DEVELOPMENT OF AN INVENTORY MODEL

We Consider an inventory system for deteriortaing items with the assumption that demand linear function of both selling price and on hand inventory. The total horizon is divided into m cycles, each of length T_i ; i = 1, 2,..., m. The cycle lengths are deminishing according to arithmatic progression. Each cycle (T_{i-1}, T_i) is divided into two parts in which the inventory changes with the rate $\lambda(s,t)=(\alpha-\beta s)+\gamma I_i(t)$, during (T_{i-1}, t_i) and during (t_i, T_i) the back order accumulates and having negative inventory due to shortages and the back orders are fulfilled.

With the assumptions and notations given in the section 2, the differential equations governing the system during the i^{th} cycle are

$$\frac{dI_i(t)}{dt}=-[(\alpha-\beta s)+\gamma I_i(t)]-\theta I_i(t)\ ;T_{i-1}\le t\le t_i, i=1,2,...,m \tag{1}$$

$$\frac{dI_i(t)}{dt}=-[(\alpha-\beta s)+\gamma I_i(t)]\ \ ;t_i\le t\le T_i, i=1,2,...,(m-1) \tag{2}$$

with the initial conditions *$I_i(t)=0$* at *$t = t_i$* for all *i=1,2,..., (m-1)*. Solving the above equations we get

$$I_i(t)=\frac{\alpha-\beta s}{\gamma+\theta}\left[e^{(\gamma+\theta)(t_i-t)}-1\right];T_{i-1}\le t\le t_i, i=1,2,...,m \tag{3}$$

$$I_i(t) = \frac{\alpha - \beta s}{\gamma}\left[e^{\gamma(t_i - t)} - 1\right]; \; t_i \le t \le T_i, i = 1,2,\ldots,(m-1) \quad (4)$$

The second replenishment time T_1 can be expressed as $T_1 = T, T_2 = 2T - w, T_3 = 3T - 3w$ and in general I general $T_i = iT - \frac{i(i-1)}{2}w, i = 1,2,\ldots,(m-1)$ (5)

The length of the ith cycle = T_i-T_{i-1} $=T-(i-1)w, (i=1,2,\ldots, m)$ (6)

But $\sum_{i=1}^{m-1}[T-(i-1)w] = H$ (7)

This implies $T = (m-1)\frac{w}{2} + \frac{H}{m}$ (8)

The ordering quantity Q_i in the i^{th} cycle (T_{i-1}, T_i) is given by

Q_i= Deterioration in the i^{th} cycle + Demand in the i^{th} cycle + Backlog demand in the $(i$-1$)^{st}$ cycle

$$Q_i = \left(\frac{\alpha - \beta s}{\gamma + \theta}\right)\left[e^{(\gamma+\theta)(t_i - t_{i-1})} - 1\right] + \frac{(\alpha - \beta s)}{\gamma}\left[1 - e^{\gamma(t_{i-1} - T_{i-1})}\right] \quad (9)$$

To find the total cost function, we consider various costs like ordering cost, purchasing cost, holding cost and the shortage cost.

For all the 'm' cycles the ordering cost = $m.C_3$ (10)

The holding cost of the inventory during the ith cycle is

$$\int_{T_{i-1}}^{t_i}\left(\frac{\alpha - \beta s}{\gamma + \theta}\right)\left[e^{(\gamma+\theta)(t_i - t_1)} - 1\right]dt \quad (11)$$

The holding cost of the inventory in the last cycle is $\int_{T_{m-1}}^{H_i}\left(\frac{\alpha - \beta s}{\gamma + \theta}\right)\left[e^{(\gamma+\theta)(H-t)} - 1\right]dt$ (12)

Using the equations (11) and (12)The total holding cost of the inventory in all the cycles is

$$\sum_{i=1}^{m-1}\left(\frac{C_1(\alpha - \beta s)}{\gamma + \theta}\right)\int_{T_{i-1}}^{t_i}\left[e^{(\gamma+\theta)(t_i - t_1)} - 1\right]dt + \left(\frac{C_1(\alpha - \beta s)}{\gamma + \theta}\right)\int_{T_{im-1}}^{tH}\left[e^{(\gamma+\theta)(H - t_1)} - 1\right]dt \quad (13)$$

The total purchasing cost for all the m cycles is

$$C(\alpha - \beta s)\sum_{i=1}^{m-1}\int_{T_{i-1}}^{t_i}\left[e^{(\gamma+\theta)(t_i - t_1)} - 1\right]dt + (t_i - T_{i-1}) + \frac{\left(1 - e^{\gamma((t_i - T_{i-1}))}\right)}{\gamma}$$

$$+ C(\alpha - \beta s)\left[\int_{Tm-1}^{H}\left[e^{(\gamma+\theta)(H_i - t_1)} - 1\right]dt + (H - T_{im-1})\right] + \frac{\left(1 - e^{\gamma((t_{m-1} - T_{m-1}))}\right)}{\gamma} \quad (14)$$

The total shortage cost for 'm' cycles = $C_2\left(\frac{\alpha-\beta s}{\gamma}\right)\sum_{i=1}^{m-1}\int_{t_i}^{T_{i_i}}\left[e^{(\gamma)(t_{i-1-t1})}-1\right]dt$ (15)

Adding various costs given in equations (10), (13), (14), (15) we get the total cost function as

$$K(m,t_i,T_i)=m\ C_3=\frac{[C_1+C(\gamma+\theta)](\alpha-\beta s)}{\gamma+\theta}\sum_{i=1}^{m-1}\int_{T_{i-1}}^{t_i}\left(e^{(\gamma+\theta)(t_i-t)}-1\right)dt+C(\alpha-\beta s)\sum_{i=1}^{m-1}\left[(t_i-T_{i-1})+\left[\frac{1-e^{\gamma(t_i-T_{i-1})}}{\gamma}\right]\right]+$$

$$\frac{[C_1+C(\gamma+\theta)](\alpha-\beta s)}{\gamma+\theta}\left[\int_{T_{m-1}}^{H}\left(e^{(\gamma+\theta)(H-t)}-1\right)dt\right]+C(\alpha-\beta s)\left[(H-T_{m-1})+\frac{1}{\gamma}\left(1-e^{\gamma(t_{m-1}-T_{m-1})}\right)\right]-C_2\left(\frac{\alpha-\beta s}{\gamma}\right)\sum_{i=1}^{m-1}\int_{t_i}^{T_i}\left(e^{(\gamma(t_i-t))}-1\right)dt \quad (16)$$

5. OPTIMAL ORDERING POLICIES FOR COST MINIMIZATION OF THE MODEL

The optimal ordering policies of the model are considered by minimising the total cost function given in equation (16). For a fixed 'm' the corresponding optimal values of t_i (i=1,2,..., (m-1)) are obtained as the solutions of the system of (m-1) equations

$$\frac{\partial}{\partial t_i}[K(m,t_i,T_i)]=0 \quad \text{for } i = 1,2,\ldots,(m-1) \text{ and } \frac{\partial^2}{\partial t_i^{\,2}}[K(m,t_i,T_i)]>0$$

For a fixed 'i' the cost function is

$$K(m,t_i,T_i)=m\ C_3=\frac{[C_1+C(\gamma+\theta)](\alpha-\beta s)}{\gamma+\theta}\left[\frac{\left(e^{(\gamma+\theta)(t_i-t)}-1\right)}{\gamma+\theta}-(t_i-T_{i-1})\right]+C(\alpha-\beta s)\sum_{i=1}^{m-1}\left[(t_i-T_{i-1})+\left[\frac{1-e^{\gamma(t_i-T_{i-1})}}{\gamma}\right]\right]$$

$$\frac{[C_1+C(\gamma+\theta)](\alpha-\beta s)}{\gamma+\theta}\left(\frac{\left(e^{(\gamma+\theta)(H-T_{m-1})}-1\right)}{\gamma+\theta}-(H-T_{m-1})\right)+C(\alpha-\beta s)\left[(H-T_{m-1})+\frac{1}{\gamma}\left(1-e^{\gamma(t_{m-1}-T_{m-1})}\right)\right]-C_2\left(\frac{\alpha-\beta s}{\gamma}\right)\left(\frac{1-e^{\gamma(t_i-t)}}{\gamma}-(T_i-t_i)\right)$$

Differentiating K with respect to t_i equating it to zero, we get

$$\frac{[C_1+C(\gamma+\theta)]}{\gamma+\theta}\left[e^{(\gamma+\theta)(t_i-T_{i-1})}-1\right]+C-\frac{C_2}{\gamma}\left(1-e^{\gamma(t_i-T_i)}\right)=0$$

expanding the exponential function and neglecting the higher order items of order $t_i^{\,2}$ we get

$$[C_1+C(\gamma+\theta)(t_i-T_{i-1})]+C-\frac{C_2}{\gamma}(\gamma(t_i-T_i))=0$$

$$t_i=\frac{V\,T_{i-1}+T_i-\frac{C}{C_2}}{V+1} \quad \text{where } V=\frac{C_1+C(\gamma+\theta)}{C_2} \quad (17)$$

This gives the optimal values of t_i for which the total cost is minimum.,Therefore $t_i-T_i=\frac{V\,T_{i-1}+T_i-\frac{C}{C_2}}{V+1}-T_i$ (18)

$$t_i - T_{i-1} = -\frac{\left(V\left(T_{i-1} - T_{i-2}\right) + \frac{C}{C_2}\right)}{V+1} \tag{19}$$

$$t_i - t_{i-1} = \frac{V\left(T_{i-1} - T_{i-2}\right) + \left(T_i - T_{i-1}\right)}{V+1} \tag{20}$$

Using the equations (17), (18), (19) and (20) in the equation (16), the total cost function is

$$K\left(m, t_i, T_i\right) = m\,C_3 + \frac{\left(C_1 + C(\gamma+\theta)\right)(\alpha - \beta s)}{6}\left\{3\left[\frac{\left(T_i - T_{i-1}\right) - \frac{C}{C_2}}{V+1}\right] + (\gamma+\theta)\left[\frac{\left(T_i - T_{i-1}\right) - \frac{C}{C_2}}{V+1}\right]^2\right\}$$

$$+ C(\alpha - \beta s)\sum_{i=1}^{m-1}\left\{\frac{V\left(T_{i-1} - T_{i-2}\right) + \left(T_i - T_{i-1}\right)}{V+1} - \frac{\gamma^2}{2}\left(\frac{V\left(T_{i-1} - T_{i-2}\right) + \frac{C}{C_2}}{V+1}\right)^2\right\}$$

$$+ \frac{\left(C_1 + C(\gamma+\theta)\right)(\alpha - \beta s)}{6}\left[3\left(H - T_{m-1}\right)^2 + (\gamma+\theta)\left(H - T_{m-1}\right)^3\right] +$$

$$C(\alpha - \beta s)\left[\left(H - t_{m-1}\right) - \frac{\gamma^2}{2}\left(t_{m-1} - T_{m-1}\right)^2\right] + \frac{C_2(\alpha - \beta s)}{6}\sum_{i=1}^{m-1}\left[3\,\frac{\left(V\left(T_i - T_{i-1}\right) + \frac{C}{C_2}\right)}{V+1}\gamma\left(\frac{\left(V\left(T_i - T_{i-1}\right) + \frac{C}{C_2}\right)}{V+1}\right)^3\right] \tag{21}$$

But $T_{m-1} = (m-1)T - \frac{(m-1)(m-2)}{2}w$ and $T = (m-1)\frac{w}{2} + \frac{H}{m}$

$H - T_{m-1} = H - (m-1)\left(\frac{w}{2} + \frac{H}{m}\right)$ (22) and $T_i - T_{i-1} = (m+1-2i)\frac{w}{2} + \frac{H}{m}$ (23)

Further $T_{i-1} - T_{i-2} = (m+3-2i)\frac{w}{2} + \frac{H}{m}$ and $T_{m-1} - T_{m-2} = (3-m)\frac{w}{2} + \frac{H}{m}$

Now Substituting these values of $\left(t_i - T_{i-1}\right), \left(t_i - T_i\right), \left(t_{i-1} - T_{i-1}\right)$ the total cost function becomes a function of two variables 'm' and 'w'. Hence denote it by *K(m,w)*.

$$K(m,w) = m\,C_3 + \left(\frac{C_1 + C(\gamma+\theta)}{6}\right)(\alpha - \beta s)\sum_{i=1}^{m-1}\left[3\left(\frac{(m+1-2i)\frac{w}{2} + \frac{H}{m} - \frac{C}{C_2}}{V+1}\right)^2 + (\gamma+\theta)\left(\frac{(m+1-2i)\frac{w}{2} + \frac{H}{m} - \frac{C}{C_2}}{V+1}\right)^3\right]$$

$$+ C(\alpha - \beta s)\sum_{i=1}^{m-1}\left[\left(\frac{V\left[(m+3-2i)\frac{w}{2} + \frac{H}{m}\right] + \left[(m+1-2i)\frac{w}{2} + \frac{H}{m}\right]}{V+1}\right) - \frac{\gamma^2}{2}\left(\frac{V\left[(m+3-2i)\frac{w}{2} + \frac{H}{m}\right] + \frac{C}{C_2}}{V+1}\right)^2\right]$$

$$+\left[\frac{C_1+C(\gamma+\theta)}{6}(\alpha-\beta s)\right]\left[3\left(H-(m-1)\left(\frac{w}{2}+\frac{H}{m}\right)\right)^2+(\gamma+\theta)\left(H-(m-1)\left(\frac{w}{2}+\frac{H}{m}\right)\right)^3\right]$$

$$+C(\alpha-\beta s)\left[H-\frac{V\left[(m-2)w+\frac{H}{m}\right]+(m-1)\left(\frac{w}{2}+\frac{H}{m}\right)-\frac{C}{C_2}}{V+1}\right]-\frac{\gamma^2}{2}\left[\frac{V(3-m)\frac{w}{2}+\frac{H}{m}+\frac{C}{C_2}}{V+1}\right]$$

$$+\frac{C_2(\alpha-\beta s)}{6}\sum_{i=1}^{m-1}\left[3\left(\frac{(m+1-2i)\frac{w}{2}+\frac{H}{m}+\frac{C}{C_2}}{V+1}\right)^2+(\gamma+\theta)\left(\frac{(m+1-2i)\frac{w}{2}+\frac{H}{m}+\frac{C}{C_2}}{V+1}\right)^3\right] \tag{24}$$

Here 'm' is a discrete variable and 'w' is a continuous variable. The optimal value of 'w' can be obtained by solving so that the total cost function is minimum for this 'w'. For a fixed 'm' from equation (24)

$$\frac{\partial}{\partial w}[K(m,w)]=\left(\frac{C_1+C(\gamma+\theta)}{6}\right)(\alpha-\beta s)\sum_{i=1}^{m-1}\left[6\left(\frac{(m+1-2i)\frac{w}{2}+\frac{H}{m}-\frac{C}{C_2}}{V+1}\right)+3(\gamma+\theta)\left(\frac{(m+1-2i)\frac{w}{2}+\frac{H}{m}-\frac{C}{C_2}}{V+1}\right)^2\right]\left(\frac{m+1-2i}{2(V+1)}\right)$$

$$++C(\alpha-\beta s)\sum_{i=1}^{m-1}\left[\left(\frac{V\frac{\left[(m+3-2i)+\frac{H}{m}\right]}{2}+\left[\frac{(m+1-2i)}{2}+\frac{H}{m}\right]}{V+1}\right)-\gamma^2\left(\frac{V\left[(m+3-2i)\frac{w}{2}+\frac{H}{m}\right]+\frac{C}{C_2}}{V+1}\right)\left(\frac{m+3-2i}{2(V+1)}\right)\right]$$

$$+\left[\frac{C_1+C(\gamma+\theta)}{6}(\alpha-\beta s)\right]\left[6\left(H-(m-1)\left(\frac{w}{2}+\frac{H}{m}\right)\right)+3(\gamma+\theta)\left(H-(m-1)\left(\frac{w}{2}+\frac{H}{m}\right)\right)^2\right]\left(\frac{1-m}{2}\right)$$

$$+C(\alpha-\beta s)\left[-\frac{V[(m-2)]+\frac{(m-1)}{2}}{V+1}\right]-\frac{\gamma^2}{2}\left[\frac{V(3-m)\frac{w}{2}+\frac{H}{m}+\frac{C}{C_2}}{V+1}\left(\frac{3-m}{2}\right)\right]$$

$$+\frac{C_2(\alpha-\beta s)}{6}\sum_{i=1}^{m-1}\left[6\left(\frac{(m+1-2i)\frac{w}{2}+\frac{H}{m}+\frac{C}{C_2}}{V+1}\right)+3(\gamma+\theta)\left(\frac{(m+1-2i)\frac{w}{2}+\frac{H}{m}+\frac{C}{C_2}}{V+1}\right)^2\right]\left(\frac{m+1-2i}{2}\right)=0 \tag{25}$$

Solving the equation (25) using MATH CAD 6.0. The optimal values w* of w can be obtained. The corresponding optimal values K*of K can be found from (24). Also the optimal values T* of T, T_i* of T_i ; i =1,2,..., m can be obtained.

The optimal ordering quantity Q^*_i of Q_i of the inventory in the i^{th} cycle where $1 \le i \le m-1$ is

$$Q_i^* = \left(\frac{\alpha - \beta s}{\gamma + \theta}\right)\left[e^{(\gamma+\theta)\left(t_i^* - T_{i-1}^*\right)} - 1\right] + \frac{\alpha - \beta s}{\gamma}\left[1 - e^{\gamma\left(t_{i-1}^* - T_{i-1}^*\right)}\right] \quad \text{where } t_i^* = \frac{T_i^* + VT_{i-1}^*}{1+V}$$

The total revenue R is calculated as

$$= s(\alpha - \beta s)H + \gamma s\left(\frac{\alpha - \beta s}{\gamma + \theta}\right)\sum_{i=1}^{m-1}\left[\frac{e^{(\gamma+\theta(t_i - T_{i-1}))} - 1}{\gamma + \theta} - (t_i - T_{i-1})\right] + \left[\frac{e^{(\gamma+\theta(H - T_{m-1}))} - 1}{\gamma + \theta} - (H - T_{m-1})\right]$$

$$+ \frac{\gamma s(\alpha - \beta s)}{\gamma + \theta}\sum_{i=1}^{m-1}\left[\frac{e^{(\gamma+\theta(t_i - T_{i_i}))} - 1}{\gamma + \theta} - (t_i - T_i)\right] \tag{26}$$

Now the total profit function of the inventory is given by $P(s, t_i, T_i)$ = Total Revenue-Total Cost

$$= \quad s(\alpha - \beta s)H + (\gamma s - (C_1 + C(\gamma + \theta)))\left(\frac{\alpha - \beta s}{\gamma + \theta}\right)\left\{\sum_{i=1}^{m-1}\left[\frac{e^{(\gamma+\theta(t_i - T_{i-1}))} - 1}{\gamma + \theta} - (t_i - T_{i-1})\right] + \left[\frac{e^{(\gamma+\theta(H - T_{m-1}))} - 1}{\gamma + \theta} - (H - T_{m-1})\right]\right\}$$

$$+ \frac{\gamma s(\alpha - \beta s)}{\gamma + \theta}\sum_{i=1}^{m-1}\left[\frac{e^{(\gamma+\theta(t_i - T_{i_i}))} - 1}{\gamma + \theta} - (t_i - T_i)\right] - mC_3 \tag{27}$$

For fixed values of m,C_1,C_2,C_3, C, s, H and q, the optimal values of w*, T* Q_i*, K* and P* are computed and presented in Table (1.1). Sensitivity analysis also conducted with tabulated values.

6. OPTIMAL PRICING AND ORDERING POLICIES UNDER VARIABLE SELLING PRICE

Here we obtain the optimal pricing and ordering policies of the inventory system under variable selling price. In the previous section (5), we considered the selling price 's' as fixed. However, in many situations, the selling price is a variable and can be fixed by developing optimal pricing policy. To obtain the optimal selling price along with the optimal ordering quantity, we maximise the total profit rate function given in equation (27) For fixed 'i' the profit function is $P(s,t_i,T_i)$

$$s(\alpha - \beta s)H + (\gamma s - (C_1 + C(\gamma + \theta)))\left(\frac{\alpha - \beta s}{\gamma + \theta}\right)\left\{\left[\frac{e^{(\gamma+\theta(t_i - T_{i-1}))} - 1}{\gamma + \theta} - (t_i - T_{i-1})\right] + \left[\frac{e^{(\gamma+\theta(H - T_{m-1}))} - 1}{\gamma + \theta} - (H - T_{m-1})\right]\right\}$$

$$+ \frac{\gamma s(\alpha - \beta s)}{\gamma + \theta}\left[\frac{e^{(\gamma+\theta(t_i - T_{i_i}))} - 1}{\gamma + \theta} - (t_i - T_i)\right] - mC_3$$

Maximising the profit rate function with respect to t_i by solving $\frac{\partial P(t, t_i, T_i)}{\partial t_i} = 0$ we got $t_i = \frac{V T_{i-1} + T_i - \frac{C}{C + \gamma s}}{V + 1}$

Substituting the values of $t_i - T_{i-1}, t_i - T_i, t_{i-1} - T_{i-1}, t_{m-1} - T_{m-1}$ (t_i-T_{i-1}), (t_i-T_i), (t_{i-1}-T_{i-1}), and t_{m-1}-T_{m-1} using the the total profit function becomes a function of *w* and *s* only and hence denote it by *P(w,s*) and is given by

P(w,s) =

$$s(\alpha-\beta s)H+\left(\gamma s-\left(C_1+C(\gamma+\theta)\right)\right)\left(\frac{\alpha-\beta s}{\gamma+\theta}\right)\left\{\sum_{i=1}^{m-1}\left[\frac{e^{\left(\gamma+\theta\left[\frac{\left[(m+1-2i)\frac{w}{2}+\frac{H}{m}\right]-\frac{C}{C_2+\gamma s}}{V+1}\right]\right)}-1}{\gamma+\theta}-\left[\frac{\left[(m+1-2i)\frac{w}{2}+\frac{H}{m}\right]-\frac{C}{C_2+\gamma s}}{V+1}\right]\right]\right\}$$

$$+\frac{\gamma s(\alpha-\beta s)}{\gamma+\theta}\sum_{i=1}^{m-1}\left\{\left[1-e^{-\gamma\left[\frac{V\left[(m+1-2i)\frac{w}{2}+\frac{H}{m}\right]+\frac{C}{C_2+\gamma s}}{V+1}\right]}\right]-\gamma\frac{V\left[(m+1-2i)\frac{w}{2}+\frac{H}{m}\right]+\frac{C}{C_2+\gamma s}}{V+1}\right\}+$$

$$\left(\gamma s-\left(C_1+C(\gamma+\theta)\right)\right)\left[\frac{e^{(\gamma+\theta)\left[\frac{(H-(m-1))\left(\frac{w}{2}+\frac{H}{m}\right)}{V+1}\right]}-1}{\gamma+\theta}\right]-(H-(m-1))\left(\frac{w}{2}+\frac{H}{m}\right)$$

$$-mC_3-\frac{C_2(\alpha-\beta s)}{\gamma^2}\left[\left[1-e^{-\gamma\left[\frac{V\left[(m+1-2i)\frac{w}{2}+\frac{H}{m}\right]+\frac{C}{C_2+\gamma s}}{V+1}\right]}\right]-\gamma\frac{V\left[(m+1-2i)\frac{w}{2}+\frac{H}{m}\right]+\frac{C}{C_2+\gamma s}}{V+1}\right]$$

We have maximised *P(w,s)* with respect to w and s to get the optimal selling price and successive reduction in cycle time by solving the equations Simultaneously we get the optimal values of w and s as w* and s* and the optimal value of the profit function P*(w*,s*). For various values of the parameters m and the costs C_1,C_2, C_3, C the optimal values of the selling price and the rate of reduction in successive cycle lengths w are computed using MATHCAD 6.0 and are tabulated in **Table.1**

Table 1: Optimal Values of the Parameters of the Model Having Shortages (Variable Selling Price)

θ	C_1	C_2	C_3	C	s	m	W	T_1	Q_1	T_2	Q_2	T_3	Q_3	T_4	Q_4	K	P
0.1	0.1	5	9	2	14.30	3	2.17	6.17	118.19	10.17	60.86	12	22.85			478.05	1435
0.2					14.64		1.43	5.43	117.54	9.43	68.85	12	35.93			520.04	1375
0.3					14.99		1.13	5.13	125.64	9.13	76.95	12	44.23			568.48	1229
	0.2				14.43		1.86	5.86	105.43	9.86	59.60	12	27.05			493.92	1286
	0.3				14.56		1.65	5.65	96.74	9.65	58.37	12	29.83			510.32	1249
		6			14.29		2.14	6.14	119.03	10.14	61.51	12	23.32			479.52	1332
		7			14.28		2.12	6.12	119.73	10.12	62.03	12	23.68			480.76	1336
			10		14.30		2.17	6.17	118.19	10.17	60.86	12	22.85			481.08	1432
			11		14.30		2.17	6.17	118.19	10.17	60.86	12	22.85			484.08	1429
			12		14.30		2.17	6.17	118.19	10.17	60.86	12	22.85			487.08	1426
				3	15.21		1.64	5.64	88.135	9.64	53.34	12	27.42			607.18	1123
				4	16.16		1.34	5.34	69.605	9.34	46.25	12	27.68			708.62	870.7
						4	1.53	5.29	92.407	9.29	55.99	11.3	28.72	12	8	449.68	1327

7. OBSERVATIONS

As the rate of deterioration increases the optimal value of the selling price is also increasing. This phenomenon is much closer to the realistic situation in the inventory of deteriorating items. As the rate of deterioration increases the wastage is more and the loss is to be shared between the seller and the buyer.

8. SENSITIVITY ANALYSIS

Sensitivity analysis has been performed to the model with variable selling price and with shortages with respect to the ordering cost C_3, holding cost C_1, unit cost C, shortage cost C_2, demand parameters α, β, γ and the deterioration parameter θ and all parameters together on the total cost of the system and the ordering quantities in different cycles. Table - 2 summarizes the results. The following data has been considered for the sensitivity analysis.

C_1 = Rs.0.1, C_2 = Rs.5, C_3 =Rs.9, C = Rs.2, α = 0.1, β = 25, γ = 1 and θ = 0.1.

From the Table. 2, the values of the ordering quantity Q_1 varies from 130.928 to 190.011, Q_2 varies from 99.461 to 140.475, Q_3 varies from 72.163 to 99.378 and the total cost varies from 615.404 to 1173 for 15% under-estimation and over-estimation of all costs and parameters under consideration.

Table 2: Sensitivity Analysis of the Model with Shortages

Variation Parametres		Percentage Change in Parametre						
		-15	**-10**	**-5**	**0**	**5**	**10**	**15**
C_3	K	886.8581	888.208	889.408	890.908	892.258	893.608	894.958
	Q_1	63.712	163.712	163.712	163.712	163.712	163.712	163.712
	Q_2	122.692	122.692	122.692	122.692	122.692	122.692	122.692
	Q_3	87.892	87.892	87.892	87.892	87.892	87.892	87.892
C_1	K	881.917	884.92	887.917	890.908	893.894	896.874	899.848
	Q_1	164.038	163.9291	163.821	163.712	163.604	163.497	163.389
	Q_2	122.8668	22.807	122.749	122.692	122.632	122.574	122.516
	Q_3	7.969	87.945	87.921	87.892	87.874	87.515	87.826
C_2	K	889.916	889.8121	890.38	890.908	891.398	891.855	892.282
	Q_1	161.1551	62.086	162.935	163.712	164.427	165.086	165.696
	Q_2	20.936	121.575	122.157	122.692	123.181	123.633	124.052
	Q_3	86.773	87.812	87.555	87.892	88.212	88.502	88.771
C	K	770.471	810.509	850.655	890.908	931.269	971.738	1012
	Q_1	166.806	165.758	164.727	163.712	162.715	161.733	160.761
	Q_2	124.715	124.029	123.354	122.692	122.038	121.395	120.769
	Q_3	89.113	88.701	88.296	87.892	87.506	87.12	86.742
θ	K	868.831	876.168	883.557	890.908	898.31	905.734	913.178
	Q_1	158.797	160.424	162.062	163.712	165.375	167.051	168.739
	Q_2	119.814	120.769	121.728	122.692	123.658	124.629	125.604
	Q_3	86.377	86.833	87.39	87.892	88.405	88.914	89.423
α	K	743.651	92.737	841.822	890.908	939.944	989.079	1038
	Q_1	135.807	145.109	154.411	163.712	173.014	182.316	191.618
	Q_2	101.777	108.748	115.719	122.692	129.662	136.633	143.604
	Q_3	72.915	77.909	82.903	87.892	92.892	97.886	102.88

β	K Q_1 Q_2 Q_3	908.579 167.061 125.2 89.695	902.689 165.945 124.364 89.096	896.798 164.829 123.567 88.497	890.908 163.712 122.692 87.892	885.018 162.596 121.854 87.298	879.127 161.48 121.017 86.669	873.237 160.324 120.181 86.10
γ	K Q_1 Q_2 Q_3	871.081 157.87 119.317 86.183	877.574 159.777 120.421 86.745	884.193 161.724 121.546 87.317	890.908 163.712 122.692 87.892	897.72 165.742 123.856 88.487	904.628 167.014 125.043 89.086	911.634 169.929 126.251 89.695
All Parametres	K Q_1 Q_2 Q_3	615.404 130.928 99.461 72.163	700.286 141.492 107.002 77.307	791.781 152.418 114.744 82.551	890.908 163.712 122.692 87.892	980.876 171.88 128.809 92.278	1111 187.439 139.204 98.893	1173 190.011 140.475 99.378

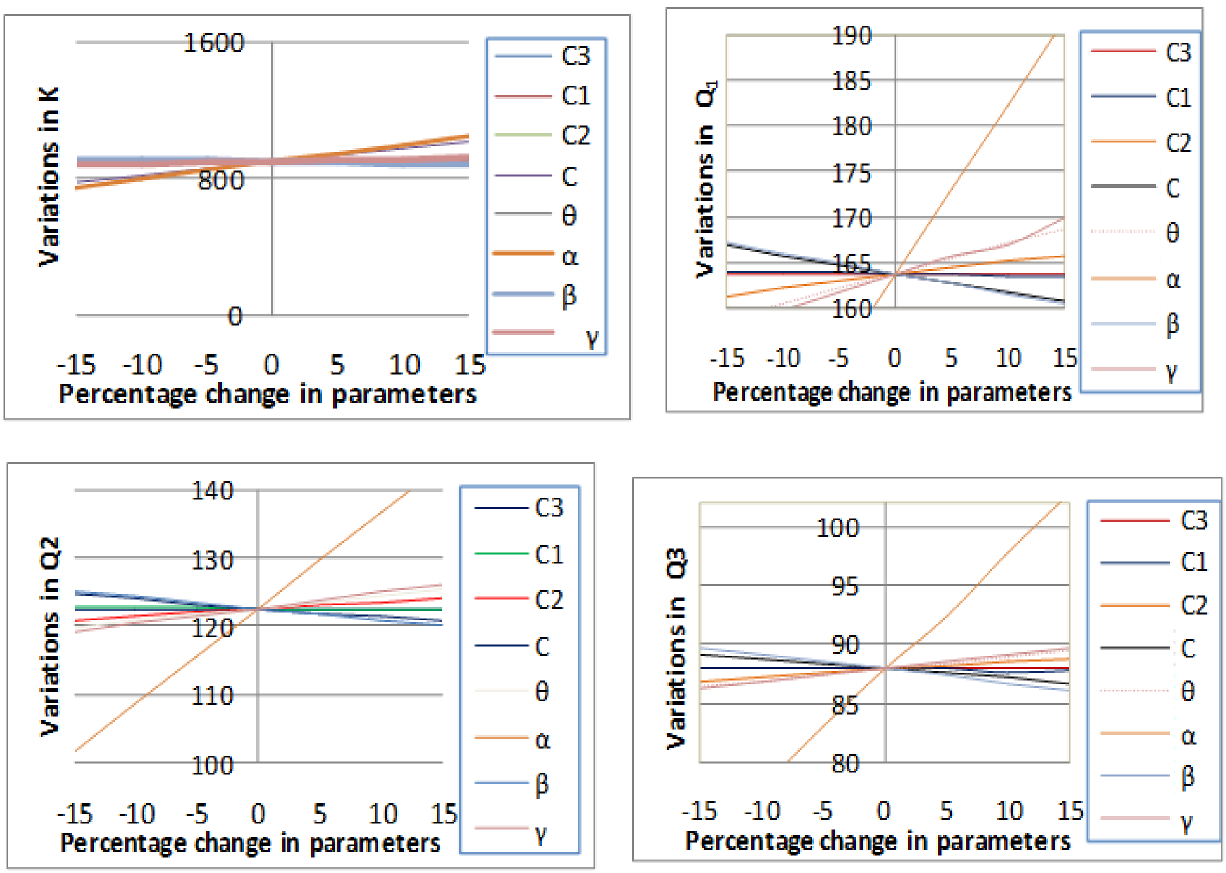


Figure 1: Graphical Representation of the Sensitivity with Respect to the Parameters of the Model with Variable Fixed Selling Price and with Shortages

9. OPTIMAL ORDERING POLICIES UNDER VARIABLE SELLING PRICE WITHOUT SHORTAGES

If the selling price is a variable then to obtain the optimal selling price along with the optimal ordering quantity, we maximise the total profit of the inventory with respect to the selling price 's' and reduction in successive cycle lengths 'w'. When shortages are not allowed then $V = \dfrac{C_1 + C(\gamma+\theta) - \gamma s}{C_2 + \gamma s} = 0$ as $C_2 \to \infty$ and $t_i = T_i$ Then the profit function

$$s(\alpha-\beta s)H + (\gamma s - (C_1 + C(\gamma+\theta)))\left(\frac{\alpha-\beta s}{\gamma+\theta}\right)\left\{\sum_{i=1}^{m-1}\left[\frac{e^{\left(\gamma+\theta\left[\frac{\left[(m+1-2i)\frac{w}{2}+\frac{H}{m}\right]}{V+1}\right]\right)}-1}{\gamma+\theta} - \left[\frac{\left[(m+1-2i)\frac{w}{2}+\frac{H}{m}\right]}{V+1}\right]\right]\right\}$$

$$+\frac{\gamma s(\alpha-\beta s)}{\gamma+\theta}\sum_{i=1}^{m-1}\left\{\left[1-e^{-\gamma\left[\frac{V\left[(m+1-2i)\frac{w}{2}+\frac{H}{m}\right]}{V+1}\right]}\right]-\gamma\frac{V\left[(m+1-2i)\frac{w}{2}+\frac{H}{m}\right]}{V+1}\right\}+$$

$$(\gamma s - (C_1 + C(\gamma+\theta)))\left[\frac{e^{(\gamma+\theta)\left[\frac{(H-(m-1))\left(\frac{w}{2}+\frac{H}{m}\right)}{V+1}\right]}-1}{\gamma+\theta}\right] - (H-(m-1))\left(\frac{w}{2}+\frac{H}{m}\right) - mC_3$$

To find the optimal values of w and s the equations $\frac{\partial P(w,s)}{\partial w}=0$ and $\frac{\partial P(w,s)}{\partial s}=0$ are solved simultaneously for the optimal values w* and s* of w and s, and for different values of the parameters and costs the optimal values of $T_1, T_2, ..., T_m$, Q_1, $Q_2, ..., Q_m$, K and P are obtained and tabulated in Table.3.

Table 3: Optimal Values of the Parameters of the Model without Shortages(Variable Selling Price)

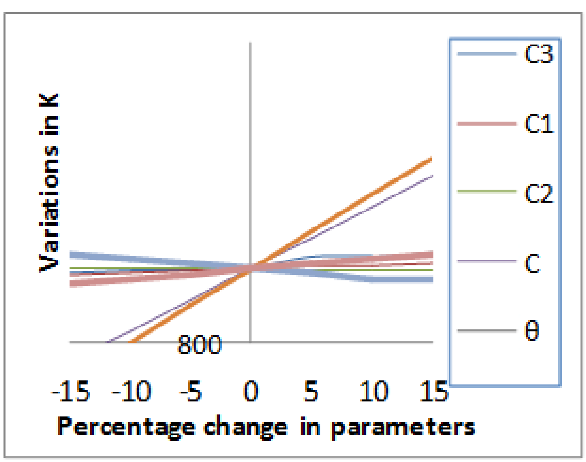
θ	C_1	C_3	C	s	m	W	T_1	Q_1	T_2	Q_2	T_3	Q_3	T_4	Q_4	K	P
.1	0.1	9	2	14.19	3	2	6	134.81	10	66.25	12	26.59			489.71	1474
.05				13.99		2.19	5.19	151.21	9.2	60.43	12	14.33			474.48	1648
.2				14.69		1.17	5.17	166.14	9.3	79.72	12	45.97			538.53	1249
	0.2			14.38		1.67	5.67	134.81	9.7	65.22	12	31.64			508.51	1386
	0.3	10		14.61		1.43	5.43	134.81	9.8	63.69	12	34.19			524.34	1315
		11		14.19		2.00	6.00	134.81	10	66.25	12	26.59			492.11	1471
		12		14.19		2.00	6.00	134.81	10	66.25	12	26.59			495.71	1468
				14.19		2.00	6.00	134.84	10	66.25	12	26.59			498.71	1465
				14.27	4	1.40	5.40	128.68	8.4	58.85	11.5	31.33	12	10.55	451.69	1367

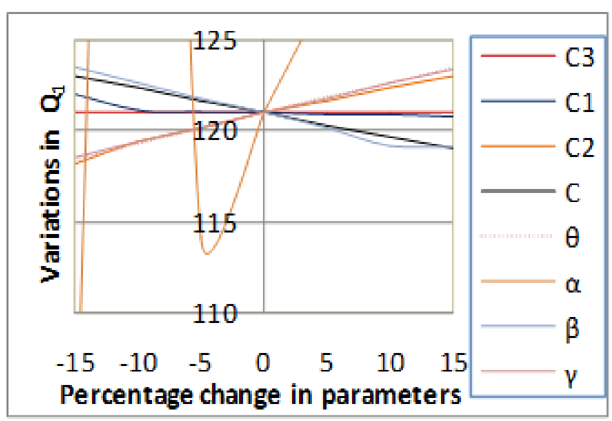
10. SENSITIVITY ANALYSIS

Sensitivity analysis has been performed to the model with variable selling price and without shortages with respect to the ordering cost C_3, holding cost C_1, unit cost C, demand parameters α, β, γ and the deterioration parameter θ and all parameters together on the total cost of the system and the ordering quantities in different cycles. Table -4 summarizes the results. The following data has been considered for the sensitivity analysis.C_1 = Rs.0.1, C_3 =Rs.9, C = Rs.2, α = 0.1, β = 25, γ = 1 and θ = 0.1

Table 4: Sensitivity Analysis of the Model without Shortages

Variation Parametres		Percentage Change in Parametre						
		-15	**-10**	**-5**	**0**	**5**	**10**	**15**
C_3	K	894.216	895.566	896.916	898.266	913.47	914.82	916.17
	Q_1	120.971	120.971	120.971	120.971	120.971	120.971	120.971
	Q_2	117.271	117.271	117.271	117.271	117.271	117.271	117.271
	Q_3	113.628	113.628	113.628	113.628	113.628	113.628	113.628
C_1	K	889.715	892.569	895.419	898.266	901.107	903.945	906.748
	Q_1	121.174	121.106	121.039	120.971	120.904	120.896	120.769
	Q_2	117.462	117.398	117.344	117.271	117.207	117.144	117.081
	Q_3	113.809	113.748	113.688	113.628	113.568	113.509	113.449
C_2	K	900.100	899.414	898.806	898.266	897.781	897.346	896.951
	Q_1	118.414	119.342	120.191	120.971	121.690	122.356	122.973
	Q_2	114.825	115.712	116.524	117.271	117.959	118.595	119.186
	Q_3	111.290	112.138	112.195	113.628	114.286	114.894	115.459
C	K	774.997	815.992	857.082	898.266	939.543	980.913	1022
	Q_1	123.065	122.355	121.658	120.971	120.295	119.631	118.976
	Q_2	119.276	118.597	117.928	117.271	116.623	115.986	115.360
	Q_3	115.549	114.898	114.258	113.628	113.008	112.399	111.798
Θ	K	877.677	884.508	891.375	898.266	905.181	912.12	919.083
	Q_1	118.450	119.287	120.127	120.971	121.818	122.669	123.523
	Q_2	114.900	115.687	116.477	117.271	118.067	118.866	119.669
	Q_3	111.402	112.241	122.883	113.628	114.375	115.125	115.875
A	K	749.757	799.258	848.762	898.266	947.769	997.273	1047
	Q_1	100.351	197.224	114.098	120.971	127.844	134.718	141.591
	Q_2	97.281	103.944	110.608	117.271	123.934	130.597	137.26
	Q_3	94.26	100.716	107.142	113.628	120.084	126.540	132.997
B	K	916.087	910.46	904.206	898.266	892.325	885.197	884.603
	Q_1	123.445	122.621	121.796	120.971	120.146	119.156	119.074
	Q_2	119.669	118.870	118.070	117.271	116.471	115.512	115.432
	Q_3	115.952	115.178	114.403	113.628	112.853	111.924	111.846
Γ	K	877.961	884.711	891.479	898.266	905.07	911.892	918.731
	Q_1	118.533	119.343	120.156	120.971	121.789	122.609	123.432
	Q_2	114.981	115.742	116.505	117.271	118.038	118.808	119.58
	Q_3	111.481	112.195	112.911	113.628	114.347	115.068	115.791
All Parametres	K	623.036	707.975	799.643	898.266	1004	1116	1243
	Q_1	98.633	105.904	113.349	120.971	128.771	135.682	145.515
	Q_2	95.738	102.752	109.929	117.271	124.779	131.431	140.833
	Q_3	92.833	99.645	106.56	113.628	120.852	127.252	136.333





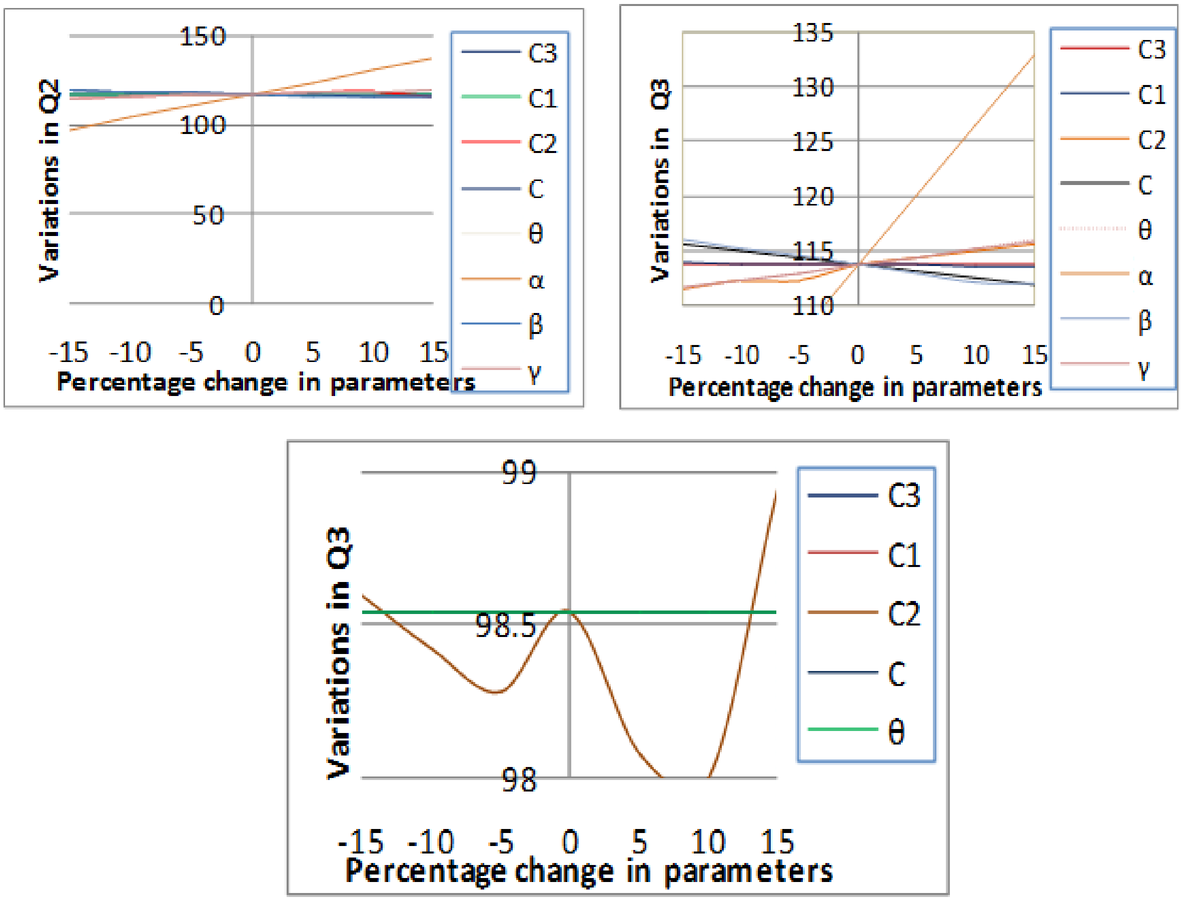


Figure 2: Graphical Representation of the Sensitivity with Respect to the Parameters of the Model without Shortages

11. SUMMARY AND CONCLUSIONS

It is observed that the policy of not allowing shortages has a significant influence on the optimal ordering policies of the model. The optimal profit of the inventory system in the model without shortages is less when compared to the model with shortages when the other parameters and costs are fixed. The optimal ordering quantity in the first, second and in the third cycle are more for the model without shortages than those of the model with shortages. The optimal selling price is more in the model with shortages than that of without shortages when the other parameters and cost are fixed. It is further assumed that the optimal profit is more in the case of without shortages when compared to the model with shortages when the other parameters and costs are fixed. From the Table.1and Table.3, it is observed that the policy of allowing shortages have tremendous effect on the optimal selling price and optimal ordering policies of the model. It is observed that the optimal selling price is more in the model with than that of without shortages, when the other parameters and costs are fixed. It is further observed that the optimal profit is more in the case of without shortages when compared to the model with shortages when other parameters and costs are fixed.

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